Adjoint-based Shape Optimization for Turbulent Convective Heat Transfer with a Hybrid RANS-DNS Approach

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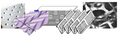
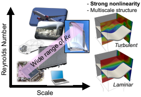
Heat Exchanger Design

Dilemma: Strong similarity between Heat transfer & Drag

=> Origin of dissimilarity
Momentum => Divergence-free vector
Energy => Conservative Scalar

e.g. Optimal control theory can achive complete dissimilar control, Yamamoto et al., J. Fluid Mech. (2013)





Shape optimization algorithm for turbulent convective heat transfer is needed.

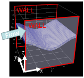
	Shape optimization strategy for turbulent flow 1) Steady approach	2) Unsteady approach
Forward	RANS Simulation (steady) – Low cost – Accuracy of turbulence model	DNS/LES (unsteady) – High cost – Highly accurate prediction
Adjoint	Steady adjoint simulation	Unsteady adjoint simulation Invalid due to the strong nonlinearity
Design	Performance is not guaranteed due to low accuracy of forward analysis.	To be established => Present study, 3)

3) DNS-RANS Hybrid Approach
"RANS-based adjoint analysis" + "Turbulent statistics from DNS"

Objective:
Development of adjoint-based shape optimization algorithm for turbulent convective heat transfer with DNS-based Eddy Viscosity & Diffusivity

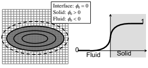
Optimization Algorithm

Target: Wavy Fin



Uniformly heated fluid is cooled by the channel walls and wavy fin

Shape Expression



Level-set Function, ϕ_0 ⇨ Shape indicator, ϕ

Optimal shape ⇔ Optimal distribution of ϕ

Governing equations & Numerical set-up

$$\frac{\partial \mathbf{u}}{\partial t} = -\nabla(\mathbf{uu}) + \frac{1}{Re}\nabla^2\mathbf{u} - \nabla p - \eta\phi\mathbf{u}$$
$$\frac{\partial \theta}{\partial t} = -\nabla(\theta\mathbf{u}) + \frac{1}{Pe}\nabla^2\theta + Q - \eta\phi\theta$$
$$\nabla\mathbf{u} = 0$$

Impose no-slip & isothermal conditions on fin by
Volume Penalization Method
e.g. Peskin C. S., *J, Comput. Phys.* **10** (1972)

$Re = \frac{u_\tau \delta}{\nu} = 200$
$Pe = Re\,Pr = 200$
δ: channel-half width
ν: kinematic viscosity
u_τ: friction velocity

[Boundary conditions]
Streamwise, x: Periodic boundary
Spanwise, z: Periodic boundary
Wall-normal, y: $(u,v,w) = \mathbf{0}$, $\theta = 0$
[Flow conditions]
Constant mean pressure gradient
Constant uniform heating of the fluid

[Domain Size]
$(L_x, L_y, L_z) = (2\delta, 2\delta, 2\delta)$
[No. of grid points]
$(N_x, N_y, N_z) = (128, 128, 128)$
[Scheme]
Spatial discret.: 2nd order FD
Time integration: RK3/CN2
Pressure coupling: SMAC

Cost Functional

"More heat transfer with less pressure drag"
=> "Maximizing the ratio of heat transfer to pressure drop: Analogy factor, A_f"

$$J = -A_f = -\frac{Nu}{fRe_b} = -\frac{\textit{Dimennsionless heat transfer coefficient}}{\textit{Dimennsionless puming power}}$$

DNS-based Turbulence Model Tuning

Time-Averaged Governing Equations

$$0 = -\nabla(\overline{\mathbf{u}}\overline{\mathbf{u}}) + \frac{1}{Re}\nabla^2\overline{\mathbf{u}} - \nabla\bar{p} - \eta\phi\overline{\mathbf{u}} - \nabla(\overline{\mathbf{u}'\mathbf{u}'})$$
$$0 = -\nabla(\bar{\theta}\overline{\mathbf{u}}) + \frac{1}{Pe}\nabla^2\bar{\boldsymbol{\theta}} - \bar{Q} - \eta\phi\bar{\theta} - \nabla(\overline{\theta'\mathbf{u}'})$$
$$\nabla\overline{\mathbf{u}} = 0$$

Red terms:
To be modeled with mean fields.

Boussinesq Approximation

Reynolds shear stress and turbulent heat flux are modeled with eddy viscosity ν_τ and eddy diffusivity α_τ

$$-\overline{\mathbf{u}'\mathbf{u}'} \simeq \boxed{\nu_\tau}(\nabla\overline{\mathbf{u}} + \nabla\overline{\mathbf{u}}^T) - \frac{2}{3}k\boldsymbol{I}$$
$$-\overline{\theta'\mathbf{u}'} \simeq \boxed{\alpha_\tau}\nabla\bar{\theta}$$

Not unique ...

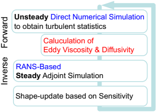
k, TKE

Dot product with mean gradient

$$-\overline{\mathbf{u}'\mathbf{u}'} : \nabla\overline{\mathbf{u}} = \boxed{\nu_\tau}(\nabla\overline{\mathbf{u}} + \nabla\overline{\mathbf{u}}^T) : \nabla\overline{\mathbf{u}}$$
$$-\overline{\theta'\mathbf{u}'} \cdot \nabla\bar{\theta} = \boxed{\alpha_\tau}\nabla\bar{\theta} \cdot \nabla\bar{\theta}$$

Uniquely determined so as to reproduce the production of turbulent kinetic energy and thermal fluctuation.

Flowchart



Unsteady Direct Numerical Simulation to obtain turbulent statistics

Caluculation of Eddy Viscosity & Diffusivity

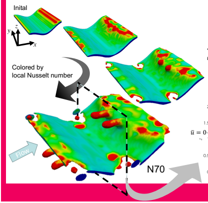
RANS-Based **Steady** Adjoint Simulation

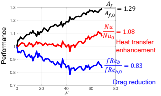
Shape-update based on Sensitivity

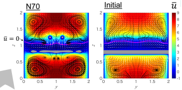
Feature of current algorithm
Level-set function
- Applicable to topology change
- Cartesian grid system allows a high accuracy scheme for turbulent simulaton.

DNS-based model tuning
- Highly accurate turbulence model

Results & Discussion







Mechanism

Drag Reduction
<= Flow rate increase by
- Cross sectional area increase
- Suppression of recirculation region

Heat transfer Enhancement
<= Secondary flow enhancement by
- Holes on fin-crest near channel side-wall
- Streamwise protrusion at the channel-center region

Summary

- New adjoint-based shape optimization algoristhm for convective turbulent heat transfer is proposed and validated with a wavy fin.
- The present algorithm uses turbulent statistics obtained by DNS for tuning of eddy viscosity and diffusion in RANS-based adjoint analysis.

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